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CONTENTS

The Nature of Space—I: THEODORE DE LAGUNA 393

A Metaphysician's Petitio: G. A. TAWNEY 407

Book Reviews:

Proceedings of the Aristotelian Society, 1920–21: H. T. COSTELLO ... 414

Journals and New Books 418

Notes and News 419

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The Journal of Philosophy

THE NATURE OF SPACE—I

Introduction

The topic of the present study is to be understood in a very restricted sense, and the essential restrictions must be made plain at the outset. Every textbook of geometry may be said to be an essay on the nature of space, that is to say, of space in the abstract—space *assumed* either as matter of fact or as matter of hypothesis. This little essay does not pretend to rival the textbooks of geometry. It does, indeed, have occasion to treat of some very elementary geometrical concepts, but not of anything more complicated than the straight line. At the same time, it is not content to assume abstract space, either as matter of fact or as hypothesis. On the contrary, its princiapl object is to exhibit the conception of space in its actual setting in experience.

However, it is not the intention to try to give an account of the whole of this setting, but only of so much of it as is strictly relevant to the *geometrical* treatment of space. With the further physical aspects of space we shall have almost nothing to do. The notion of mass and the notion of a measurable duration will not enter into our discussion—not to speak of the notions of force and energy. But a very important aspect of the meaning of *distance* consists in its relations to mass, duration, *etc.* No man can throw a baseball two hundred yards; and no man can run a mile in three minutes. From the geometrical point of view, the kilometer has no properties distinct from those of the millimeter; and if all the distances in the world were multiplied by a million there would be no change at all. From the physical point of view, even the doubling of all lengths and distances would have a very profound effect upon the world, if bodies on the surface of the earth continued to fall about sixteen feet during the first second.

The reader will, therefore, realize that in restricting consideration to the geometrical conception of space, we very greatly simplify our problem, and at the same time seriously limit the possible value of any solution which we may reach. However successful we may be, we shall have taken but a single step toward the larger systematic knowledge of space, which can only be obtained as an essential part of the knowledge of the general character of the physical world.

There are two motives for this restriction of the inquiry. One is purely personal: the narrowness of the writer's knowledge and competence. This, if it stood alone, would be a better reason for not publishing at all than for publishing the result of a truncated investigation. But there is a further motive in the belief that within the limits that are thus laid down a tolerably complete and satisfactory solution of the problem can be given.

There is this also to be said. While results, such as are here given, are seriously limited in their range, they have a place of their own in the system of science; and at the present juncture their importance may be very great. How far Professor Einstein has gone into detail in this matter, I do not know; but it is clear from his account of the use of "measuring-rods" that some such theory as that here set forth is presupposed by him in his two-fold theory of relativity. Professor Whitehead, who is the author both of an independent account of the electromagnetic theory of relativity and of a revision of Newton's theory of gravitation, has set forth very fully his conception of spatial order and measurement in two remarkable volumes, *An Enquiry into the Principles of Natural Knowledge* and *The Concept of Nature*. That conception is radically inconsistent with the analysis here given.

Certain of the differences between Professor Whitehead's account and the present one may be specified as follows. His account starts with *events;* the present account starts with a certain class of *objects*, namely, *physical solids*. Among the events which he assumes, some are unlimited in three dimensions; here only finite solids are assumed. The assumed unlimited events have, with reference to their fourth (temporal) dimension, a definite *shape;* they are analogous to the three-dimensional space between parallel planes; the solids from which the present account starts are of all possible shapes, but no one shape is assumed as given. Most important of all, however, is the fact that Professor Whitehead makes the conceptions of the point, line, and surface—and the solid, too, for that matter—as well as of linear order, length, and distance, logically dependent upon the intersection of moments of different time-systems; while to me this appears to be a most unfortunate distortion of the actual system of relationships.

There is a deeper ground of difference, however, which ought not to pass unnoticed. Professor Whitehead's work is based upon a certain theory as to the nature of experience, which seems to me to be extremely doubtful. He takes his start from certain data, which, as he believes, are given to us in sensuous experience. My own point of departure is in the behavior of things toward one another, as we manipulate them.

Among Professor Whitehead's incontestably important contributions is his method of "extensive abstraction." In the precise form in which he has described it, it was not available for my purposes. But, in an appendix, I have used a simplified, and in some respects strengthened, form of the method for the definition of the point, the line, and the surface as sets of solids; and I have at the same time showed how the method is related to my own assumptions.

The main body of the essay is in two parts. The first is mathematical; but, lest this should affright any modest philosophical reader, let me hasten to add that the mathematics is of the very simplest character—quite as easy as the easiest pages of the first book of Euclid. (In the notes there are passages that may make rather more difficult reading; but these are not essential.) The object of this part is to lead to as clear as possible a conception of certain of the more fundamental geometrical entities and relations.

The second part is physical. Its object is to determine the empirical foundations of geometry (so far as these may properly be held to lie within the limits of a physical, rather than a psychological, inquiry). The point and the peculiar relation between points, which, in the mathematical part, figure as primary assumptions in terms of which explanation is to be made, are here themselves the goal that is to be attained. The physical part of the inquiry is thus directly supplementary to the mathematical. The former owes its problem to the outcome of the latter.

At the end of the second part, an opportunity arises for making a suggestion the import of which extends far beyond the subject of these pages. This opportunity is afforded, first, by the close similarity of the conception here reached of the principles of geometry, to the conception of physical principles long ago advanced by Galileo, as well as to that recently advanced by Poincaré; and, secondly, by a further analogy to the general principle of empirical science—the uniformity of nature. I shall take advantage of this opportunity to the extent of offering the briefest possible comment.[1]

[1] With the exception of the appendix dealing with the method of extensive abstraction, this essay has lain in manuscript for the last five years. An earlier, more general study, *The Nature of Primary Qualities*, was published in the *Philosophical Review* for September, 1913 (Vol. XXII, pp. 502ff.). A second paper, *On the Distinction between Primary and Secondary Qualities*, dealing more fully with certain outlying epistemological questions, appeared in this JOURNAL for February 28, 1918 (Vol. XV, pp. 113ff.).

The Fundamental Concepts of Geometry

Every reader, the memory of whose youthful days has not faded out completely, will recall that in the study of elementary geometry one sets out from a body of concepts that are accepted as too simple and clear to need definition, and that in terms of these primary concepts one defines all other concepts that belong to the science. The number of these assumed "indefinables" is usually very great; but, since they are not plainly listed and set apart, they may easily seem to be far fewer than a careful search would show.

It has long been an enterprise of mathematicians, dating particularly from the researches of Leibniz, first, to make an accurate list of the assumed concepts of geometry, and, secondly, to reduce their number to the absolute minimum by defining all that can be defined. This enterprise has in recent years been rewarded with a large measure of success. It has been found possible to base the concepts of geometry upon two "indefinables," one of which is an entity, the other a relation.[2]

What was not anticipated in the old days is the fact that a considerable freedom of choice is possible in choosing the indefinables. The entity chosen is, indeed, usually the *point*. But for the relation there are three important alternatives to choose from, giving rise to three fairly distinct types of geometrical system: projective geometry, descriptive geometry,[3] and metrical geometry.

(i) For projective geometry the indefinables are now usually "point" and a certain relation between three points called "to be

[2] This last statement must not be misunderstood. Geometry makes the freest use of the concepts of formal logic, as well as of the more complex mathematical concepts (particularly those of arithmetic) which are definable in terms of logical concepts. Such terms as "if—then," "and," "or," "not," "any," "such as," "identical with," *etc.;* as well as "one," "two," "as many," "more," "twice as many," *etc.*, are thickly sprinkled all over the pages of our geometries. Some of these must, in any system of logic, be accepted as indefinable. When, therefore, we speak of the two indefinables of a system of geometry, we mean the two *peculiar* indefinables, assumed in addition to the omnipresent concepts of logic.

To guard against misplaced verbal criticism, it may be added that there is an interpretation of geometry, according to which its "indefinables" are really defined—namely, by the "axioms," or "postulates," from which the demonstrations of the science proceed. The argument of the present chapter is unaffected by these considerations.

[3] The usage of this term is unfortunately inconsistent. In the following pages it will be used to denote the type of geometry for which measurement is a wholly secondary matter, but which assumes from the outset—as projective geometry does not—the conception of the *order* of three points in a straight line. *Cf.* B. Russell, *The Principles of Mathematics*, p. 382 and Chap. XLVI; L. Couturat, *Les Principes des Mathématiques*, p. 142 and pp. 159ff.

collinear." In common language this relation would be expressed by saying that the three points were in one straight line. The same mode of expression may, indeed, be used by the mathematician; but he has then three indefinables instead of two: namely, "point," "line" (or "straight line"), and "to be in." It is simpler to start with "point" and "collinear," and to define the straight line as a peculiar set of points; that is to say, as the set of all the points that are collinear with two distinct points, together with the two points themselves.

We may pause here to emphasize the fact that collinearity is a relation between *three* points. The relations with which logic has for the most part dealt are relations between two terms; and from this fact a wide-spread prejudice has arisen, to the effect that all relations are confined to two terms. But the case of collinearity itself is clear evidence to the contrary. *Any two* points are connected by a straight line; but when this is true of three points it constitutes a certain definite relation between them.

It is true, that instead of regarding collinearity as a relation between three points, we may equally well regard it as a *quality of a collection* of three points.[4] That is, of course, because the relation of collinearity is symmetrical. If A is collinear with B and C, it is collinear with C and B; and furthermore B is collinear with A and C, and C is collinear with A and B. Such a relation can always be regarded as a quality that is predicable of the collection as a totality. Instead of saying that Henry is a cousin of Stephen or that Stephen is a cousin of Henry, I may equally well say that Henry and Stephen are a pair of cousins.

(ii) For descriptive geometry the approved indefinables are "point" and "to be between." This latter is, again, a relation between three points. It would be expressed in ordinary language by saying that one point was in a line with two others, and between them; but in descriptive geometry it is accepted as a simple, inexplicable *datum*.

The between-relation differs from the relation of collinearity in being symmetrical only with respect to two of the three points and asymmetrical with respect to either of those two points and the

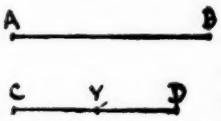
[4] The properties that we ascribe to things are of two sorts: first, those that we may ascribe to a single thing and, secondly, those that may only be ascribed to two or more things conjointly. "To be brave" is an example of the first sort; "to be a cousin of" is an example of the second sort. These two sorts of properties may, with essential fidelity to tradition, be distinguished as "qualities" and "relations." It should be observed, however, that in accordance with this usage, while "to be a cousin of" is a relation, "to be a cousin of Peter" is a quality; for we may affirm it of a single subject.

third point. If A is between B and C, then it is between C and B; but B is not between A and C, nor is C between A and B.[5] Accordingly the between-relation is not interpretable as a quality of the collection of three points. The peculiar position of the middle point can not be expressed in any such way. The relation may, however, be regarded, not as a relation between three terms, but as a relation between two; namely, the middle point and the collection consisting of the other two points.

(iii) Metrical geometry has been less successful than the other two types of geometry in finding appropriate and serviceable indefinables. Metrical geometry is what most of us mean when we use the word without an adjective—the science as set forth by Euclid and his direct successors. In fact the expression "metrical geometry" is etymologically a crude tautology. But the science of space has been so extended during the last century, that its original subject of spatial measurement has become for it an altogether secondary interest. The historical fact remains, that metrical geometry is the original type, and that projective and descriptive geometry are comparatively recent specialized developments of conceptions first reached by the metrical mode of approach. The less satisfactory condition of the foundations of metrical geometry must, therefore, be deeply regretted by those who wish to understand the place of geometry in human experience.

Since the time of Leibniz, the generally preferred indefinables for metrical geometry have been "point" and a *class of relations* between two points, called "distances." It is assumed that the same relation of distance that subsists between two points may also subsist between two other points. As a matter of fact the distance between two points never appears in a proposition except as it is in some way compared with the distances between other points.

The unsatisfactoriness of these indefinables depends upon the awkwardness of the procedure by which it is necessary to introduce the notion of one distance being *greater than* another distance. The following device is as simple as any:



"The distance AB is greater than the distance CD" means that there exists a point Y, such that the distance CY is the same as the distance DY, and such that there is no point X such that the distances AX, BX, and CY are all the same.[6]

[5] Sometimes a certain class of exceptions is admitted: it is assumed that any point is "between" itself and any other. (In the same way it may be assumed in projective geometry that any point is collinear with itself and any other point.) This is to be regarded as a merely verbal matter, to be determined by convenience of terminology.

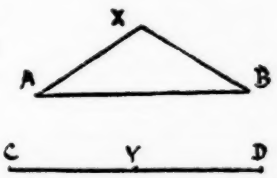
It is to be observed that this device does not enable us to define the relation "greater than" except on the implied assumption that space is of at least two dimensions. If we limit space to a single straight line, then, if the distances *AB* and *CD* are not identical, *AB* is *necessarily* greater than *CD* by the above definition; for, on that supposition, if the distances *CY* and *DY* were the same, we could *never* find a point *X* such that *AX*, *BX*, and *CY* were identical.

If we assume that space is of only one dimension, we can, indeed, define the relation "greater than" with reference to any two distances that are *commensurable* with each other. We say, for example, that if the distances *AB*, *BC*, *CD*, and *DE* are all the same, then the distance *AE* *contains* the distance *AB* four times. If, in the same way, the distance *XY* contains *AB* three times, then, since (in the arithmetical sense of the terms) four is *greater than* three, we say: "*AE* is greater than *XY*."[7]

How unfortunate it is that the relation "greater than" should be made to depend upon multidimensionality or else upon commensurability, is seen from the consideration of *time*. Time we are in the habit of conceiving on the analogy of the straight line, as a space of one dimension. Now, in the case of time, we make no scruple to assume that one interval may be longer than another, altogether independently of the question of commensurability. Why should we have to look outside of the straight line itself in order to give a general meaning to the proposition that one of its segments is greater than another?[8]

When we reflect upon the matter, we can see that there is at

[6] This figure illustrates the fact that if *AB* were less than *CD* a point *X* that satisfied the given requirement could easily be found. It must, however, always lie outside of the line *AB*.



[7] It might be supposed that this mode of definition could be extended to incommensurable distances by the method of limits; but this appears to be impossible. One can not even give a meaning to the expression, "*B* lies between *A* and *C*," unless *AB* and *BC* are commensurable.

[8] It is, of course, possible to assume "greater-than" (in the sense of a relation between two distances) as a third geometrical indefinable, and this is sometimes, openly or covertly, done. But such a procedure, *if it is not unavoidable*, has this demerit: it puts a limitation upon the understanding of the subject-matter of the science—it effectually prevents a maximum clearness. It is true that a high degree of simplicity is thereby facilitated; but in the study of the principles of mathematics simplicity is too dearly purchased at the expense of clearness. Each additional undefined term that might have

bottom a very good reason why the distance-relation makes a very poor conception upon which to base the science of spatial measurement. As we have already remarked, one distance-relation never appears alone in a proposition, but only in comparison with another distance-relation. Geometry takes cognizance of no peculiar properties of any particular distance. Hence the distance between two given points, when it is considered by itself, apart from any relation to the distances between other points, is as nearly as possible an *empty* concept.

This may, perhaps, be made clearer by an illustration. Suppose a man cast ashore upon a desert island, where among the useful articles that he has saved from the wreck he finds a pair of compasses and a straight-edge, but no yard-measure and nothing of any known ratio to the yard. Can he reproduce the yard? He can not. The yard, like every other measure of length, is a *conventional* unit. To be a yard long means to be just as long as something else accepted as being a yard long. If a standard of reference is wanting, the yard disappears.

The suggestion thus arises that for the construction of a metrical geometry the fundamental relation ought to be a relation between *four* points, or, if you please, between two "pairs" of points —collections of points consisting of two each.[9] One such relation is "to be just as far apart as." Another is "to be farther apart

been dispensed with means so much more superficiality—it means that the labor of analysis that ought to be done has not been done.

In this connection the remark may be made, that the number of assumed axioms—provided these are mutually independent and are sufficient to prove what they are expected to prove—is almost entirely indifferent. If anything, the greater number of axioms is preferable, for this may show a finer analysis; but this does not appear to be necessarily the case. There is a good deal of popular misunderstanding on this point, because of the fact that so much endeavor has been directed by mathematicians upon the elimination of *superfluous* axioms—axioms that could really be proved from the remaining assumptions. It needs to be clearly understood that if none of the axioms of a given set is superfluous—if they are all mutually independent—their number is, generally speaking, a matter of no importance.

[9] To be perfectly precise, one must admit the limiting case in which the "pair" consists of but one and the same point "twice considered." This, however, is a topic that belongs to general logic. The pair "*A* and *B*" may there be defined as the class (or collection) of which *A* is a member and *B* is a member, and which has no other members. It is not specified, and it is not implied, that *A* and *B* must be distinct.

The classical geometry conceived of this limiting case in a characteristically different way. Instead of "considering the same point twice over," it admitted *coincident* points. There is no reason why we today should not adopt a similar procedure; and for the purposes of this study there would be some advantages in doing so. I have preferred, however, to conform to the prevailing fashion as far as possible.

than." There are several reasons for preferring the latter, of which only one is strictly pertinent in this place; namely, that the choice of the former amounts to practically the same thing as the assumption of distance itself as an indefinable class of relations.[10]

One other reason, however, is of too great methodological importance to be denied mention. The assertion of a relation such as "to be just as far apart as" is generally based, not on positive evidence, but on the absence of negative evidence. If we are dealing with physical objects, and A and B are farther apart than C and D, we can in many instances attest this fact with a high degree of assurance. As, however, this relation *approaches* the condition where A and B are just as far apart as C and D, the certainty of our judgment decreases; and a condition is finally reached where all that we can say is that we see no further difference. Generally speaking, propositions asserting the relation "to be farther apart than" are of a higher degree of probability than those asserting the relation "to be just as far apart as." What, therefore, the proposition, that A and B are just as far apart as C and D, actually *means* in the system of science is that, within the limits of our observation, A and B are not farther apart than C and D, and C and D are not farther apart than A and B.

It is very striking, how, when the relation, "to be farther apart than," is taken as fundamental, the difficulties dwindle away. This can best be shown by exhibiting a specimen series of definitions. It is true that, from the mathematical standpoint, the definitions that occur in mathematics are matters of merely verbal significance. They are statements that certain newly presented symbols may, at will, be substituted for certain combinations of previously presented symbols. Nevertheless, it is in the definitions that the analysis of the complex subject-matter of the science most clearly appears.

The series of definitions given in the following pages extends as far as the introduction of the notions which are fundamental to projective and descriptive geometry. Farther than that we need not go; for the "defining-value" of these notions is well known, and that of our own indefinables will have been shown to be at least

[10] It may be recalled that in Professor Veblen's well-known account of the foundations of elementary geometry *two* indefinable relations are employed. In addition to the relation of *congruence*, which is essentially the same as "to be just as far apart as," he introduces the relation of *order* between three points, which is essentially the same as the between-relation. (I say "essentially," because in each case there may be an unimportant technical difference.) The resulting system is, therefore, not purely metrical, but is a mixture of metrical and descriptive elements; and it labors under the disadvantage which was pointed out in a previous note.

as great. Our task, then, will be to make clear what, from the metrical standpoint, is meant by the *order of points in a straight line.*

INDEFINABLES

I. Point.

II. To be farther apart than.

Points are to be represented by capital letters. Different letters need not indicate distinct points.

DEFINITIONS

I. If A and B are not farther apart than C and D, and C and D are not farther apart than A and B, then A and B are said to be *just as far apart as* C and D.

II. The *distance* AB is the class of pairs of points that are just as far apart as A and B.

This definition calls for some comment. Instead of defining a distance as a *class* of pairs of points, we might define it as the *distinguishing property* of that class. That is to say, we might define the distance AB as the property of being just as far apart as A and B. This would be in closer accord with our common-sense notion of distance. But, as students of logic well know, propositions about classes, and propositions about the distinguishing properties of classes, run parallel to each other. The two sorts of propositions represent two different ways of regarding the same facts—in *extension* and in *intension,* to use the traditional terms. It is the general custom of mathematicians to prefer the extensive treatment; and there is no reason why we should do otherwise.

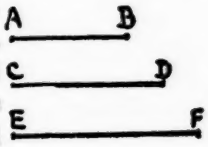
The definition of a distance as a class has this consequence: that we shall have to speak of *the same distance,* not of *equal* (or *equivalent*) *distances.* According to our definition, if A and B are just as far apart as C and D, the distance AB and the distance CD are identical. If we had chosen to define a distance as a property, the case would be different. The property of being just as far apart as A and B is not identical with the property of being just as far apart as C and D. The relation between these properties is that they *imply each other,* or are equivalent.

It is further to be remarked that the distance AB may be defined as the (symmetrical) relation subsisting between *two* points, by reason of the fact that they are just as far apart as the points A and B. This definition has nothing in particular to recommend it. But it is worth while to reflect upon it a little, because of the way in which it brings to the surface the unfitness of the two-term distance relation to serve as one of the bases of geometry.

III. If A and B are farther apart than C and D, the distance AB is said to be *greater than* the distance CD.

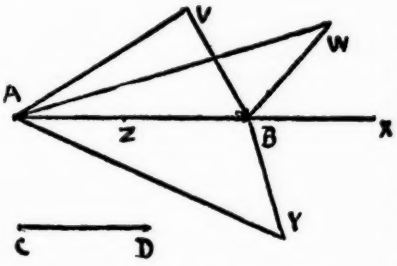
Here it is to be observed that the phrase "to be greater than" is defined in a particular technical sense: namely, as it is applied to distances. In arithmetic a relation having the same name is defined with reference to real numbers; and it is one of the first tasks of metrical geometry to justify this ambiguous use of the words by showing that its own greater-than relation has the same formal properties as the arithmetical relation. For it is on this identity of the formal properties of the arithmetical and the geometrical greater-than, that the theory of spatial measurement rests.

IV. The distances AB, CD, and EF are said to be *compatible*, if there exist the points, X, Y, and Z, such that XY is the same as AB, YZ is the same as CD, and XZ is the same as EF.



This notion of compatibility is the only one in the present set that is not familiar to common sense. We do not absolutely need it here, and it is introduced only because it enables us to define in relatively simple terms what is meant by the sum of two distances.

V. The *sum* of the distances AB and CD is a distance compatible with them, and greater than any other such distance if such there be.



In the figure let BV, BW, BX, BY, and BZ all be the same as CD. Then (by Definition IV) AV, AW, AX, AY, and AZ are all compatible with AB and CD. One of the five, namely AX, is greater than any of the others; and it is, in fact, greater than any other distance compatible with AB and CD. We call it the sum of AB and CD.

Here we must make the same remark about the use of the term "sum" that was made above with regard to the term "greater than." From arithmetic we know what the "sum" of two real numbers means. Metrical geometry has to show that the sum of two distances has the same formal properties that belong to the sum of two real numbers. This is very easily accomplished.

VI. If A, B, and C are distinct, and AC is the sum of AB and BC, B is said to be *between* A and C.

VII. If either A is between B and C, or B is between A and C, or C is between A and B; A, B, and C are said to be *collinear.*

VIII. The *straight line* AB is the class of points collinear with A and B, together with A and B.

Attention must now be called to a few characteristics of this series of definitions. In the first place, the definitions are exceedingly simple. If the reader will glance over the whole series of eight, he will see that they follow one upon another in the most natural and obvious fashion. When once the proper indefinables are chosen, the definitions are almost inevitable—not because others to take their place can not be found,[11] but because it would require a good deal of ingenuity or of industry to think of them, while these fairly suggest themselves. The only feature that smacks in the least of artificiality is the notion of "compatibility," and that only on account of the novel term that is used. The problem was, how to combine two distances so as to get their sum. Is not the obvious answer that the outside points shall be as far apart as possible? At any rate, if the definitions are not inevitable, they are at least easy—so easy that the whole set can be grasped by the mind almost without effort, in a single movement of the attention.

If the reader has studied projective geometry, and in particular the projective definition of distance, he will not hesitate to admit that the metrical definition of collinearity is incomparably simpler.

[11] An example of an alternative method is suggested by Professor C. V. Huntington, in *Mathematische Annalen*, Vol. 73, pp. 529f. Professor Huntington takes as his indefinables "sphere" and "is contained in." A "point" is defined as a sphere in which no other sphere is contained. "The point B lies between the points A and C," is then explained as meaning: No sphere X exists, in which A and C are contained, but not B.

In terms of our own indefinables, a sphere is a class of points whose distance from a certain point is not greater than a certain distance. For one sphere to be contained in another, means that every point of the former is a point of the latter. Accordingly, Professor Huntington's definition of the between-relation becomes: No point X exists such that there exists a distance YZ, such that BX is greater than YZ, and neither AX nor CX is greater than YZ, This is obviously more complex than is necessary; so we may put it: No X exists, such that BX is greater than both AX and CX.

Professor Huntington's device suggests others that to him, with his peculiar indefinables, were not available. For example, "B is between A and C" may be explained: If X exists such that AX is the same as CX, this distance is greater than BX. Or we may first explain "A, B, and C are collinear" as meaning: No X exists such that AX, BX, and CX are the same. And we may then distinguish the outer points, say by their greater distance from each other.

In any case, we may proceed to explain "AB is the sum of CD and EF" as meaning: X exists such that it is between A and B, and AX is the same as CD, and BX is the same as EF.

In the second place, the definitions imply nothing as to the number of dimensions in space. Space may equally well have one, two, three, or more dimensions, and these definitions will still hold good. Substitute the word "instant" for "point," and "interval" for "distance," and drop the last two definitions as superfluous, and the account applies perfectly to time. The like is true of the metrical principles which we shall shortly have to consider: they are all equally applicable to space of any number of dimensions.

It has sometimes been held by mathematicians that projective, descriptive, and metrical geometry presuppose one another in that order: that projective geometry is logically prior to descriptive geometry, and both of these to metrical geometry. Logicians, of course, are not at present inclined to take much stock in the notion of logical priority, except as it is conceived with reference to some particular deductive system. It has been found that there is a surprising amount of freedom in the choice of indefinable terms and indemonstrable propositions—how much we do not know. What is logically prior in one construction may be logically posterior in another. Of an absolute order of priority we know nothing.

But for our present purpose, since we have in view an inquiry into empirical foundations, the really important question of priority as between the three types of geometry is as to which gives the simplest conception of the nature of space. In this respect, metrical geometry has an incontestable and enormous advantage. The direction taken by the history of the science is here the path of least resistance. Starting from the theory of spatial measurement, we find spatial order comparatively easy to understand. Starting from the arrangement of points in lines, we find spatial measurement an intrinsically and unavoidably abstruse subject.

Our first task is accomplished. We have seen what, in metrical terms, a linear order means. But the meaning of our assumed indefinables is itself set forth in the postulates which we adopt, and becomes more and more explicit as the consequences of these postulates are successively unfolded, that is to say, as the system of geometry is constructed. It is far beyond the purpose of the present study to attempt the construction of even the foundations of a geometry. It is, indeed, not difficult to set forth a list of principles—only about sixteen are needed—from which the Euclidean geometry can be deduced. But to provide and insure the mutual independence of the principles, so that they may serve as the postulates of a mathematical system, is a task calling for a very special competence and training.

However, I have thought it well to present in this place a state-

ment of certain metrical principles, which, so far as they go, are independent of one another, and which are sufficient as premises for the demonstration of the most important properties of the between-relation. I trust that they will be of service in throwing light upon the empirical considerations that are to follow.[12]

Points, as before, are denoted by capital letters, and different letters need not indicate distinct points.

1. If A and B are farther apart than C and D, C and D are not farther apart than A and B.

In other words, "to be farther apart than" is an asymmetrical relation.

II. If A and B are farther apart than C and D, and E and F are not farther apart than C and D, A and B are farther apart than E and F.

In combination with the preceding, this assures us that if A and B are farther apart than C and D, and if C and D are farther apart than E and F, A and B are farther apart than E and F. That is to say, "to be farther apart than" is a *transitive* relation.

It also follows that the relation "to be just as far apart as" is transitive.

III. If A is not identical with B, then, for all values of X, the distances AB and XX are distinct.

This is used in demonstrating the proposition that the distance XX is constant, and is in fact the *zero-distance;* that is to say, that if XX be added to any distance YZ, the sum is YZ.

IV. If A, B, C, and D be each any point, X exists such that BX is identical with CD, and AX is the sum of AB and CD.

This assures us that any distance and any distance have a sum. It also assures us of a part of what we mean by the "uniformity" of space.

V. If AC is greater than AD, and BC is the sum of BA and AC, BC is greater than BD.

From this it follows that if AC is greater than AD, and XY is any distance, the sum of AC and XY is greater than the sum of AD and XY.

VI. If A and B are not identical, and if BC is not greater than BD; and if AD is the sum of AB and BD, and if AC is the sum of AB and BC; then BD is the sum of BC and CD.

This provides that any two distances shall have a difference. The last three principles, or their equivalents, are necessary to show that

[12] It should be remembered that we assume from general logic that the relation of co-membership in a pair is symmetrical with respect to the two members. Accordingly we assume without question that "the point A and the point B" means the same pair as "the point B and the point A."

the relation "to be greater than" is strictly analogous to the greater-than relation between real numbers.

Assuming these six principles to be true, we can easily prove the associative law for the addition of distances. (The commutative law is in this system a mere identity.) We can also prove the following series of propositions (using the expression (XYZ) to mean "Y is between X and Z"):

(i) If (ABX) and (ABY), and BX is identical with BY, X is identical with Y.

(ii) If (ABX) and (ABY), and BY is greater than BX, then (BXY) and (AXY).

(iii) If (AXB) and (AYB), and AX is identical with AY, X is identical with Y.

(iv) If (AXB) and (AYB), and AY is greater than AX, then (AXY) and (XYB).

(v) If (XAB) and (ABY), then (XAY) and (XBY).

(vi) If (AXB) and (ABY), then (AXY) and (XBY).

And from these six propositions we may derive the following generalization, which establishes the complete determination of the straight line by any two of its points:

If A, B, C, and D are distinct, and if A, B, and C are collinear and A, B, and D are collinear, then A, C, and D are collinear.

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A METAPHYSICIAN'S *PETITIO*

OF those who interest themselves to-day in the general problem of the method of experience, the greater number approach the subject in the light of experience alone. That is to say, they criticize experience without *a priori* presuppositions as to its nature. For example, they observe the processes of scientific investigation as a parent observes the breathing of her child or a physiologist the functions of the brain. They examine the industrial and commercial activities of human beings, their political strategies and recreative sports, with the meticulous care of a psychologist interested in the phenomenon of fatigue. The artistic tastes, the religious rituals and the poetic creations of people are as interesting to the methodologist as the process of metabolism to a biologist. Above all he hugs to his bosom the transformations of attitude and emotion through which reflective life passes out of one and into another of these phases of experience, believing that therein especially is ultimately to be found the synthesis that endows civilization

with integrity, the filament of spirituality that binds together the associated life of the world.

The methodologist postulates nothing but this teeming life, and that he approaches on his knees, as becomes the student of it. He does not pretend to know *a priori* that it is all a symphony played on the motifs of matter and mind: neither does he presume to know prior to observing the fact that it is not: he will see what he shall see, and to report what he has found he will use whatever conception-aids the subject beautifully fits as a glove fits the hand.

What he everywhere finds is customs and the straining of strong men to alter them, an act that is both habitual and accommodative, or rather a system of such acts rooted in the past and moving forward into an as yet problematical future. Nowhere does he find absolute gaps in the moving whole: on the contrary, human experience appears to be a processional continuum. Individuals succeed, or they fail, in utter obliviousness to the disparity between a substance that thinks and another that is extended. If one's hates are accompanied by right-hand spiral motions of the molecules of his brain and his loves by left-hand spiral motions, the whirligigs are amusing only; one is intent on his objectives, alike whether these be material or spiritual.

The methodologist notes these facts, and he too feels indifferent to metaphysical dualisms. What abstractions, however hoary, can compare in fascination with the youth and vigor, the concrete richness and teeming verve of life's artistry? In reporting his findings, the methodologist must use the language of logic, psychology and perhaps cosmology; but when he speaks of bodies he is thinking of a system of habitual activities in process of reformation; when he speaks of consciousness he is thinking of the moment of reformation itself; when he refers to objects he has in mind controlling contents of present experience that mean something not now existent but interesting and to be experienced; when he mentions the future he probably has in mind in most instances, not some distant thing that does not now exist, but the present experience of expectation and purpose that characterizes all real knowing. *Datum* and *ideatum* are not for him pebbles in the loaf of experience but moments in a continuous process.

Now comes one with a metaphysic. "Ah," he notes, "Methodologist speaks of present body and of past or future existences that are not present at all. He is therefore a dualist, probably an interactionist that knows not his own mind." Or, "He denies the existence of what I call consciousness. He is therefore a materialist ashamed of himself, a materialist ashamed to accept the implica-

tions of his own language." Mr. Lovejoy thus finds pragmatism materialistic. We do not presume to speak for all pragmatists: just possibly some of them would not assent to our present thesis: but the theory of knowledge of which Dewey and his pupils are the living exponents in this country is fundamentally a methodology, not a metaphysic, and reading metaphysical meanings into their words is a *petitio* of obvious character.

In an article in this JOURNAL (Vol. XIX, p. 6), Lovejoy is replying to an article of Bode and quotes from the latter, "A careful inventory of our assets brings to light no such entities as those which have been placed to our credit. We do not find body and object and consciousness, but only body and object." We may suppose Bode's meaning to be, "We find only organized habitual responses in process of reformation and a controlling content of present experience that means something-to-be-found-or-achieved." By way of criticism Mr. Lovejoy solemnly warns the reader that this is materialism and proceeds to refute materialism by observing, "Upon the materialistic hypothesis practical reflection itself is nothing but a motion of matter; if 'bodies and (physical) objects' are the only factors involved in 'intelligence,' it should be possible to describe the phenomenon called 'planning' wholly in physical terms —*i.e.*, in terms of masses actually existing, of positions actually occupied, of molar or molecular movements actually occurring, *at the time when the planning is taking place.*"

Thus by following the association of ideas in his own mind, by jumping with certain associations of the word "body," the metaphysician passes out of the universe of pragmatic methodology into the universe of mechanistic cosmology with an alacrity as amazing as it is alogical. Incidentally, let it be noted, as evidence of Mr. Lovejoy's desire that the reader shall follow him closely, that the word "(physical)" in parenthesis is not in the original text of Bode's article at all.

Again, we read on page seven of Mr. Lovejoy's article, "Thus in fixing his attention especially upon 'intelligence' in its practical aspect, the pragmatist is brought face to face with that type of experience in which the empirical presence of non-physical entities and processes is, perhaps, more plainly evident than in any other." If Mr. Lovejoy had written, "that type of experience in which the empirical presence of what a certain metaphysical dualist regards as non-physical entities and processes," we should have no reason to complain of his logic; but in that case there would have been no point to his criticism. Again, on page seven, "He (the pragmatist) is primarily interested, not in the question how we can know an

external, coexistent object, but in the question how one moment of experience can know and prepare for another moment. It is in short to what I have elsewhere named intertemporal cognitions that his analysis is devoted; it is by man's habit of looking before and after that he is chiefly impressed. Now to look before and after is—as my previous paper pointed out—to behold the physically non-existent."

As a matter of fact, intertemporal cognition, as Mr. Lovejoy means the phrase, is not at all what the methodologist is supremely interested in. Intertemporal cognition is cognition of a specific type, such as, "Yesterday I made an engagement that I shall fulfill tomorrow"; and the methodologist is far from being so naïve and inconsequential as to say that all intelligence is cognition of this type. On the contrary his conception of cognition holds true of this type of cognitions, if it holds at all; and Lovejoy simply begs the question he is ostensibly discussing when he thus states, or pretends to state, the methodologist's doctrine. What the latter is emphasizing is a fact noted, we believe, by every good book on psychology written within the last forty years, *viz.*, the fact that all cognitive activities are teleological functions. Lest anyone construe teleology transcendentally, as has been done so often, the methodologist describes teleology in terms of actual experience, and that involves mention of the time consumed by any process or function of actual experience. There exists, however, a vast difference between the time consumed by a cognitive function and the time content of intertemporal cognitions, a difference that every psychologist recognizes. The former is part of the form of cognition in general, the latter is content in a specific class of cognitions. Mr. Lovejoy confuses the specific cognition of temporal relations, the habit of looking before and after, with the prospective, purposive character of all cognition.

Of course the methodologist means that temporal cognition is as purposive as any other type of cognition. Observation of the time cognitions of small children has convinced the present writer that the child's first experience of time relations is his experience of enforced waiting while his nurse is coming to relieve his discomfort or bring him food. Especially when he wakens before the time for his feeding, and lies there suffering while the hands of the clock move slowly to the hour, he is becoming conscious of duration. When children first begin to talk, their judgments of futurity take on a semblance of accuracy and completeness months before their judgments of pastness do. Tomorrow's picnic excursion and the experiences of next Sunday, when the little one will be taken to

Sunday School, are fairly definite concepts long before he has any definite idea of what happened yesterday; and it is most interesting to observe how very slowly the conception of the day before yesterday begins to function in his experience. From the very start his time experience is thus relative to his needs and desires, while clock-time, the time of the mathematician, the even flowing continuum of duration, is a late abstract construction of social experience and is everywhere relative to the need to measure or otherwise conceive time, *i.e.*, to the need of organizing "the present" in a conceptual control of present activity. Quite contrary to Mr. Lovejoy's assertion, we should say the pragmatist is logically committed to a more continuous and profound interest in the past than is any other living philosopher. For history is a study of "how man came to be as he is and believe as he does,"[1] while conceptions of what man is and believes are thoroughly teleological. An historical consciousness, a consciousness that history is here and now in process of being made, is the first essential of intelligence. It is the first prerequisite of effective participation in community life. Such is the pragmatist's emphasis on knowledge of the past.

Mr. Lovejoy finds that Bode, whose article is in part a reply to an earlier one of Lovejoy himself, does not stick to the point. "Throughout most of his paper, then, Professor Bode, instead of looking at the evidence offered for this conclusion, which he ostensibly rejects, appears to fix his gaze on another object altogether." No wonder! Two men standing back to back and gazing at different points on the horizon say, the one, "That mountain is wooded to the top," and the other, "No, that mountain is bald." It is amusing to read in Lovejoy's article, "After careful study of his paper, I remain in some doubt whether he holds that pragmatism implies materialism or not." He need not be in doubt as to this point, for Bode distinctly writes, "The question what is real is absolutely sterile." The real problem, to Bode, is the problem of method. In his most metaphysical mood he would probably name his doctrine neither idealism nor materialism, neither intellectualism nor empiricism. He would probably name it experiencism,—if only there were such a word. As for realism, if that can be said to consist in treating a subject matter in a consistently objective manner without *a priori* presuppositions, he is a realist. However, as a science of the method of experience pragmatism is privileged to abstract from the problems of metaphysics, except as the metaphysician's own method of procedure presents itself as a problem.

It would be a work of supererogation to trace in detail the

[1] James Harvey Robinson.

many instances of this fallacy of begging the question in Mr. Lovejoy's *Pragmatism versus the Pragmatist,* an essay that appeared in a volume entitled *Critical Realism.* Here, however, it appears in greater variety than in the article referred to above. Here, for example, Mr. Lovejoy flings question-begging epithets about with redundant profusion. Here the rule of some lawyers, that when you have no case it is good to abuse the opposing counsel or their brief, is conscientiously followed. Here a fictitious question which is in fact a case of many questions, the presumption of proposing to decide what kind of a metaphysician the pragmatist is, is unblushingly propounded. The work of deliberately trying to make the pragmatist say things he never meant to say, of trying to make him say things that Mr. Lovejoy thinks he ought to have meant to say, and of then pretending to evaluate his doctrine by the fictitious and often fantastic results, is here arduously pursued; but it is not what one would call helpful criticism. It is as futile a procedure as trying to decide how many angels can skip around on the point of a pin.

For example, on page 46, after quoting from Dewey a statement of the pragmatist's notion of experience as "a matter of functions and habits, of active adjustments and readjustments, of coördinations and activities, rather than of states of consciousness," the article continues, "Here we have an explanation which seems to swing our interpretation of the pragmatist's position wholly over to the realistic side—and indeed to the neo-realistic side. He appears in this passage as an ardent adherent of what has been named . . . 'pan-objectivism'—as one who denies the existence of states of consciousness altogether." As if the words of Dewey could logically be construed to be a denial of the existence of states of consciousness *in the pragmatist's sense of the word!* As if indeed the first essayist of both *Studies in Logical Theory* and *Creative Intelligence* meant to say anything of the kind! Pan-objectivism affirmed one term of a traditional epistemological dualism, that of consciousness and the object, and tried to deny the other term, tardily reckoning with the fact that the terms of that dualism must be accepted or rejected together. It is absurd to say that Mr. Dewey is doing that, and yet the article in question proceeds for some pages in a fatuous attempt to criticize his theory as though he were. We can be sure that Dewey rejects Lovejoy's conception of consciousness; but it is presumptuous to assert, without even attempting to state the pragmatist's conception, that that is a denial of the existence of states of consciousness altogether.

The confusion already referred to of time-judgment with the prospective and purposive character of all judgment is even more

elaborate in the article we are now discussing. It is here a veritable dust-cloud enveloping the entire argument. "A more significant error, and one, as I think it possible to show, which is inconsistent with a true instrumentalist logic, is Mr. Dewey's Limitation of the 'knowledge-experience' *exclusively* to forward-looking thoughts," (p. 52). "This formula . . . manifestly tells only half the story, at best. It ignores the patent empirical fact that many of our 'meanings' are retrospective—and the specifically 'pragmatic' fact that such meanings are indispensable in the planning of action," (p. 53). "The pragmatist . . . manifests a curious aversion from admitting that we have knowledge, and 'true knowledge,' about the past. I have already cited from Mr. Dewey a formal definition of 'knowledge' which excludes from the denotation of the term everything except judgments of anticipation" (p. 63). The citation referred to is, I presume, the second note on page 52. But all of this exposition of instrumentalistic pragmatism misrepresents its subject, owing to its author's failure to recognize the difference between the teleological reference and effect of every genuine cognition and the mathematical time in which it transpires.

An illustration may make the matter clear. A young man is at work in the same room with me: he raises his head and looks about: he is thinking, as indeed I am, that the room is close: he notes that the windows are all closed and opens two of them. He first perceived the close air of the room, a cognitive experience based on years of past experience in similar situations, and a cognitive experience that meant for him the need of a livelier circulation of air in the room. He then perceived the closed windows-capable-of-being-opened, rose and opened two of them. To me, looking on sympathetically, the perception of the close air led to the perception that windows could be opened, and that, in turn, referred to events about to be affected by my companion's action. But to him the two perceptions were simply perceptions: they were through and through purposive. He returned to his chair with no consciousness that time had elapsed since his attention was interrupted by organic sensations of stuffiness. What he perceived was (1) the close air of the room and (2) the windows about to be opened; but to me, looking on, both perceptions involved memories of the past and ideas of the future. One need not, unless indeed he will to do so, confuse my onlooker's analysis of the processes of my friend's experience with the actual contents of those experiences; and any one who wills to confuse these two things is deliberately guilty of what might well be termed *the* metaphysician's fallacy. If such an one reads into the methodologist's description of

such experiences his own metaphysical meanings, he is guilty of the metaphysician's *petitio*.

Much is said by Mr. Lovejoy about Mr. Dewey's phrase "present-as-absent" for the purpose of convincing the reader that it denotes a time-cognition; but no student of the psychology of time-cognition is convinced by these many animadversions. The term time-cognition denotes several different sorts of cognition. At the lowest it denotes an immediate sense of duration, of interruption; (2) it denotes the cognition of intervals or periods that must elapse ere the end of a desire is attainable; (3) it denotes a sense of the values of familiar experiences with no necessary consciousness of the pastness of those experiences; (4) it denotes an explicit consciousness of intervals of past experience; and (5) it denotes the highly abstract, prodigiously useful, public thing which we have called clock-time, mathematical time with its evenly flowing continuity marked by the recurring transits of a star, by the flow of sand in an hour glass, by the uniform rate of motion of hands round the dial of a clock, by the mathematical definition of continuous manifolds, and so forth. Until the metaphysical critic of pragmatism recognizes that genuine cognitions may involve any of these meanings of the term, or none of them, his criticisms are not likely to be helpful to the methodologist. The idea that psychology has nothing to say about time-cognition that concerns the metaphysician or the epistemologist is so deeply embedded,—the idea that psychology deals solely with a morass of subjectivity that philosophy is too pure to behold,—that it is difficult for some to understand the viewpoint and method of instrumentalism. Mr. Lovejoy fails to understand not only Mr. Dewey but the whole methodological approach to the problem of knowledge, and to understand that approach is, by your leave, the first indispensable step toward a real logic. Mr. Lovejoy finds the pragmatist to be a subjective idealist, a pan-objectivist, materialist, a dualistic interactionist, and above all an errorist; but the reader will search his criticisms in vain for even an approximately adequate exposition of the doctrine he is discussing.

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BOOK REVIEWS

Proceedings of the Aristotelian Society, 1920–1921. New Series, Vol. XXI. London: Williams and Norgate. Pp. iv + 246.

Part of the strength of the excellent series of papers which make

up the present volume is due to the contributions of the American delegates to the Oxford Congress. Professor W. P. Montague presented before the Aristotelian Society (meeting of December 6, 1920) a paper entitled "Variation, Heredity and Consciousness," embodying suggestions and hypotheses concerning the relation of life and mind to the realm of physical energy. He calls his argument "a mechanist answer to the vitalist challenge." In so obscure a field any hypothesis is worthy of a trial, provided it is at all capable of verification. The present reviewer finds it hard to decide whether or no the subtle analogies to which Professor Montague appeals do present sufficient possibilities of verification. Thus he compares biological variation to vector addition in mechanics, which gives from old elements a sum which is both pertinent and novel. It is a pretty analogy, but who shall say whether it is more? Furthermore, vector addition has already itself given certain mechanistically minded students of physics considerable occasion for perplexity, as for instance, addition of accelerations. And certainly the mechanist philosopher with a single-track mind would be a bit distressed by Professor Montague's remark that "Nature is not stone-blind . . . she is an artist who works as she goes." The next analogy, which claims indeed to be a full identity, is between heredity and higher space-time derivatives, accelerations of accelerations. The comparison is said to illustrate how a great complexity of promised movement can be resident in a single particle, or small group of particles of matter, which group is at the moment apparently quiescent, like an upward thrown stone at the top of its flight. So far the analogy holds good, but the reviewer can not make clear to himself how this explains anything more about heredity. To him, the very conception of an organism, a real unit in nature, going its own way, not merely self-preservative, but using, and in a measure dominating, its environment, seems inconsistent with strict mechanism—though, for that matter, no better explained by the ineffable mysteries of vitalism. In the third part of the paper, Professor Montague again takes up, in modified restatement, his earlier identification of consciousness with potential energy, an identification, it must be said, of the mysterious with the inscrutable. Yet the comparisons instituted are certainly tantalizing, and leave the reader wondering whether in the future some new road of investigation may not open up through this region—for it is a matter of fact that there are relationships of some sort between energy and thinking, and Professor Montague's guess is not the worst that has been made. Nevertheless, the members of the Aristotelian Society may well have been left with a state of

considerable stress and confusion inhabiting their own tanks of potential energy after the first hearing of this rather bewildering paper.

Professor J. E. Boodin's paper on "Cosmic Evolution" is less disturbing to customary ideas. His discussion centers about the opinions of L. J. Henderson, the physiological chemist (*e.g.*, his *The Fitness of the Environment*), and Henry F. Osborn, the biologist (in his *The Origin and Evolution of Life*). But back of this is a wider philosophic view: "We must learn that the cosmos is the true unit of reality," and further, "There must be an eternal hierarchy of levels in the universe." It would be hard to dispute that statements so general were not in some sense true.

Professor R. F. A. Hoernlé might perhaps also be claimed as an American contributor to this volume. His "Plea for a Phenomenology of Meaning" will receive the endorsement of all students in that subtle region. The reviewer would venture to suggest that Charles Peirce's cryptic classification of signs may best be interpreted as: (*a*) indices, or using what is naturally conjoined with another as the sign of it, as palm-trees in the desert are a sign of water; (*b*) icons, or signifying through resemblance, as a portrait or a map; and (*c*) signs whose original nexus with the thing symbolized has been obscured through use, so that the relation is now arbitrary and so needs an interpretant "who remembers what it meant." I point with my finger, it is an index; I draw a picture in the air to convey my meaning, it is an icon—in either case the whole world understands; I speak a word, it is what Peirce calls a symbol, and only my own people can interpret me. This third class is doubtless rather ill represented by the word "symbol." Professor Hoernlé is well advised in again calling attention to the distinction, so notable in the case of language, between the indicative and the expressive function of signs.[1] Perhaps the third class above could be revised, still with genuine regard to Peirce's own intention, to include all those signs whose most marked characteristic is this marriage of expression of intent and designation of fact. Language is here the typical case. To the extent to which the dictum is true—and it is only partly true—that "the real is particular, but thought is of the universal," this third class is also the marriage of the universal and particular. Indices move in the realm of particulars only. You can make general statements about indices, but each several index is, as such, individual and unique in itself and

[1] The author leaves one perplexed whether his terminology follows Husserl or Meinong here (p. 85, note). Surely Meinong's usage is preferable: I *express* my own opinions and feelings, I *indicate or designate* an external fact—and every sentence I speak does both at once.

in its reference. Icons are indeed in the realm of the universal, for a portrait designates an individual only in so far as that individual has ceased to be unique and is duplicated in the portrait itself. But the user of pictures does not clearly discriminate the universal, and picture language or imitative gesture is therefore but the beginning of true language. In true language the universal and the particular have each their distinctive part. Every dictionary word stands for a universal, yet through language we do manage to discuss the particular facts of this particular world round about us. But the other aspects of language must not be forgotten either, if we are to have an adequate theory. Language is, as Professor Hoernlé well insists, a double revelation; my language is a revelation of the world I know, but also it is a revelation of me. On the other hand, let us not forget that language is social, it is a revelation to somebody and needs an interpreter. It brings minds together, but it leaves them curiously isolated. For what does really pass across? Black marks visible on paper; quiverings of the air. Out of such things we each of us, a lonely worker, build a world, wherein we nevertheless meet our fellows and come to a knowledge of self and of others. Professor Hoernlé is right in saying we need to study these queer facts more closely.

Perhaps the reviewer, through personal interest in the problems raised, has given too much space to what our American delegates told the English about philosophy. Certainly he read with more than usual interest certain of the other papers, notably the Symposium on "The Character of Cognitive Acts." The most striking proposition here is that of Mr. G. E. Moore, to the effect that in all cognition, however simple, a particular, as for instance a sense datum, is subsumed under a universal, and that there is no other characteristic, such as the existence of a knower or an act of knowing, which is thus uniformly found. This thesis suggests that Mr. Moore is working towards a new theory of mind, and we await with interest its further development. The other symposium contributions offer little that is new, though they are acute and hold the attention.

The reviewer was less impressed by Dean Inge's Presidential Address, "Is the Time Series Reversible?" than he usually is by what the Dean writes on philosophy, though no one could help being amused by the parting shot at Einstein, "I feel in my bones that this prophet of relativity is not likely to be a true friend to Platonism." Mr. C. A. Richardson, in "The New Materialism," expresses his distaste for most of the New Realists, because they have done away with what a certain sophomore student of philosophy once called "the myself part of me." If the present reviewer got little

out of Miss Oakeley's paper on Professor Driesch, this is probably the reviewer's own fault, though it may be partly attributable to Professor Driesch.

There remain two logical papers. That by Mr. F. C. S. Schiller, "On Arguing in a Circle," is one of his chronic attacks on the Absolute, but it will probably awaken neither the Absolute nor his minions. Mr. Schiller has never equalled elsewhere his one really admirable logical paper, his contribution to Singer's *Studies in the History and Method of Science* (1917). Miss Dorothy Wrinch contributes a somewhat over-ambitious paper, at least as regards its title, "On the Structure of Scientific Inquiry." It contains many good things in matters of detail. Especially interesting is her treatment of what she calls "true analogy," or identity of form of solution in problems from otherwise different fields. One earlier passage deserves quotation in full. "The domain of logic in science is overwhelmingly wide and correspondingly difficult. No student who has any knowledge of the development of science can deny this. And yet the paramount importance of logic is very seldom realized and the study of it is quite often avoided and only infrequently undertaken by those who seek to make contributions to our knowledge of the world. Professor D'Arcy Thompson in the wonderful epilogue to *Growth and Form* (1917) hints at the æsthetic glories of such a treatment of the world, in words which are full of hope and encouragement, and should earn for him the gratitude of all logicians."

H. T. COSTELLO.

TRINITY COLLEGE, HARTFORD.

JOURNALS AND NEW BOOKS

SCIENTIA. January, 1922. *Les sciences grecques et leur transmission. I. Splendeur et décadence de la science grecque* (pp. 1–10): J. L. HEIBERG (Kjöbenhavn).–Rapid historical sketch of Greek science. *The Origin of Binary Stars* (pp. 11–22): J. H. JEANS (Cambridge, England).–Concludes that some binaries are due to fission, others to independent nuclei in an original nebula. Interesting for its exposition of the methods by which these results were reached. *La contribution que les divers pays ont donnée au développement de la biologie* (pp. 23–36): MAURICE CAULLERY (Paris).–Largely a comparison of German scientific work with that of other countries, claiming to find in Germany excellent organization of research, but methods inclining towards a twisting of facts to fit *a priori* theories. *La question sociale* (pp. 37–46): *Vilfredo Pareto* (Lausanne).–Surveys the history of the conflict of social classes and

the problem of property, a many sided but rather confused summary, that hardly does its author justice. *La paix est-elle une paix anglo-saxonne?* (pp. 47–56): EDOUARD GUYOT (Rennes).–How the difference of interests has led France and England apart since the war—an interesting French view of English policies. *Les effets de la guerre sur la proportion des sexes dans les naissances* (pp. 57–62): F. SAVORGNAN (Messina).–The author takes it as proved that in the countries hardest hit by the war, the proportion of male births increased. He attributes this to the diminution of total births, with its resultant better condition of the mothers, so that fewer males were still-born. *Reviews of Scientific Books and Periodicals.*

Endara, Julio. José Ingenieros y el Porvenir de la Filosofía. Second edition. Buenos Aires: Agencia General de Librería. 1922. Pp. 100.

Rueff, Jacques. Des Sciences Physiques aux Sciences Morales: Introduction a l'étude de la morale et de l'économie politique rationelles. Paris: Félix Alcan. 1922. Pp. xx + 202. 8 fr.

NOTES AND NEWS

RESPONDING TO THE STIMULUS

To the Editors of the JOURNAL OF PHILOSOPHY:

The only reason I have to offer for taking the liberty to reply to Dr. Hunter's letter to the anti-behaviorists (in the issue of May 25), though it is together with a score of others that my name appears on his list, is to inform him that there are more than two behaviorists in this world, as I hope to show in my forthcoming little book, *Psychology and Behaviorism;* and that in his hunt for what appears to him an *avis rara* he has allowed some of the best representatives of the species, like E. B. Holt, E. A. Singer and Max Meyer, perhaps too B. Bode and Mrs. De Laguna, either to slip in among the antis or to elude his aquiline eye altogether.

I heartily agree with him in his refusal to recognize the new systems of behaviorism that are continually being put on the market as genuine products, but if he asks us introspectionists (and most of the anti-behaviorists he draws up in his formidable list are anything but introspectionists) to do a little *self-analysis,* may we not ask him *en revanche* to return the courtesy and explain *objectively* why so many objectivists in psychology should give him the impression that they are paranoiacs seeing "an enemy in everyone not an anointed introspectionist" and detecting a "danger in all objective

study''? And were these ''anointed introspectionists'' really so fearful of the inroads of behaviorism, would they be advocating new systems of behaviorism rather than eschewing the slightest association with the radical movement and employing the term *behaviorism* only in criticism? Dr. Hunter implies that every psychologist who is not a behaviorist *à la* Watson—I suppose in accordance with the famous dictum ''He who is not for me is against me''—must be an anti-behaviorist, but since he has found only two behaviorists, Watson and Weiss, he himself, by implication, must be classed as an anti—and now who will hunt the hunter?

Our interrogator is apparently given to paradox, for after admitting that he has followed ''your papers during the last ten years with keen interest and much profit'' he rather ungratefully asks ''Why do you write so much?'' May I answer for my part that it was this polygraphy—if I may use the word in an archaic sense—which has enabled me to undertake the very task which Dr. Hunter was anxious to have performed, and to answer at considerable length his questions: ''Who are the behaviorists?'' and ''Have you ever brought together a bibliography of this topic for the last decade?''

A. A. Roback.

Harvard University.

Dr. Edwin G. Boring, professor of experimental psychology at Clark University, has been appointed associate professor of psychology at Harvard University. Dr. Herbert S. Langfeld has been promoted from an assistant professorship to associate professorship at Harvard.

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The Word Integration, and a Few Remarks on the Paleontology of Words. WILLIAM E. RITTER.
Book Reviews. Journals and New Books. Notes and News.

Volume XIX. No. 11. May 25, 1922

Mr. Russell's Psychology. F. C. S. SCHILLER.
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The Verification of Standards of Value. THOMAS MUNRO.
Book Reviews. Journals and New Books. Notes and News.

Volume XIX. No. 12. June 8, 1922

Realism without Monism or Dualism—I. JOHN DEWEY.
The Differentiating Principle of Religion. H. C. ACKERMAN.
Intelligence and Intellect. A. A. ROBACK.
Book Reviews. Journals and New Books. Notes and News.

Volume XIX. No. 13. June 22, 1922

The New Materialism. JAMES BISSETT PRATT.
Realism Without Monism or Dualism—II. JOHN DEWEY.
Book Reviews. Journals and New Books. Notes and News.

Volume XIX. No. 14. July 6, 1922

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The Length of Human Infancy in Eighteenth Century Thought. ARTHUR O. LOVEJOY.
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Bi-weekly. Edited by F. J. E. Woodbridge and Wendell T. Bush.

Archives of Psychology—Sub-Station 84, N. Y.; Archives of Psychology.
Subscription $5. 500 pp. ann. Founded 1906. Ed. by R. S. Woodworth.
Published without fixed dates, each number a single experimental study.

Journal of Abnormal Psychology and Social Psychology—Boston;
Subscription $5. Richard G. Badger. Edited by Morton Prince.
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Psychological Clinic—Philadelphia; Psychological Clinic Press.
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Journal of Experimental Psychology—Princeton, N. J.
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